

About these notes:

Realization process, in a sense, means any deducible procedure that we use to rationalize and solve problems. In mathematics, especially for existence questions, a realization process often appears in the form of an iterative procedure or a differential equation. For years researchers have taken great effort to describe, analyze, and modify realization processes. Nowadays the success is especially evident in discrete numerical algorithms. On the other hand, the use of differential equations to issues in computation has been found recently to afford fundamental insights into the structure and behavior of existing discrete methods and, sometimes, to suggest new and improved numerical methods. In some cases, there are remarkable connections between smooth flows and discrete numerical algorithms. In other cases, the flow approach seems advantageous in tackling very difficult problems.

These notes, summarizing my limited understanding and ongoing research on the subject of *continuous realization methods*, were prepared for lecture presentations at the Academia Sinica in Taiwan, the Australian National University in Canberra, and various other universities during my sabbatical leave in 1996.

In this expository series the emphasis is on using differential equation techniques as a special continuous realization process for problems arising from the field of linear algebra. The matrix differential equations are cast in fairly general frameworks of which special cases have been found to be closely related to important numerical algorithms. The main thrust is to study the dynamics of various isospectral flows. This approach has potential applications ranging from new development of numerical algorithms, including VLSI implementation, to theoretical solution of open problems. Various aspects of the recent development and application in this direction will be discussed in the sequel.

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