Inverse Eigenvalue Problems: Theory and Applications

A Series of Lectures to be Presented at IRMA, CNR, Bari, Italy

Moody T. Chu
(Joint with Gene Golub)

Department of Mathematics
North Carolina State University

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Preface

An inverse eigenvalue problem concerns the reconstruction of a matrix from prescribed spectral data. The spectral data involved may consist of the complete or only partial information of eigenvalues or eigenvectors. The objective of an inverse eigenvalue problem is to construct a matrix that maintains a certain specific structure as well as that given spectral property.

Inverse eigenvalue problems arise in a remarkable variety of applications, including system and control theory, geophysics, molecular spectroscopy, particle physics, structure analysis, and so on. Generally speaking, the basic goal of an inverse eigenvalue problem is to reconstruct the physical parameters of a certain system from the knowledge or desire of its dynamical behavior. Since the dynamical behavior often is governed by the underlying natural frequencies and/or normal modes, the spectral constraints are thus imposed. On the other hand, in order that the resulting model is physically realizable, additional structural constraints must also be imposed upon the matrix. Depending on the application, inverse eigenvalue problems appear in many different forms.

Associated with any inverse eigenvalue problem are two fundamental questions – the theoretic issue on solvability and the practical issue on computability. Solvability concerns obtaining a necessary or a sufficient condition under which an inverse eigenvalue problem has a solution. Computability concerns developing a procedure by which, knowing a priori that the given spectral data are feasible, a matrix can be constructed numerically. Both questions are difficult and challenging.

In this note the emphasis is to provide an overview of the vast scope of this fascinating problem. The fundamental questions, some known results, many applications, mathematical properties, a variety of numerical techniques, as well as several open problems will be discussed.

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Moody T. Chu
Raleigh, North Carolina

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