Structured Low Rank Approximation. In a noiseless time-domain signals comprising k components of exponentially decaying sinusoids can be identified by a Hankel matrix H of rank k. The measurement A obtained from, for example, *in vivo* NMR (Nuclear Magnetic Resonance on living objects) often is of full rank because random noises have been added inevitably [2]. The challenge then is to retrieve as much information as possible about H from the observed A. It is known that the truncated singular value decomposition (TSVD) will produce the best low rank approximation [48, 49]. Nevertheless, this TSVD of A generally is no longer Hankel. Thus far, a popular method used by engineers to retrieve a Hankel low rank approximation is to alternate projections between the manifold of rank$-k$ matrices and the space of Hankel matrices. This so called Cadzow's algorithm [9, 10] has been claimed to be "a cleansing process whereby any corrupting noise, measurement distortion, or theoretical mismatch present in the given data set (namely, A) is removed." Recently, the PI and his colleagues have discovered numerical evidence using other approaches that Cadzow's algorithm usually does *not* give rise to the optimal approximation [38]. The impact of this finding is significant because the structured low rank approximation is also needed in other areas of signal processing application, including model reduction [60], speech encoding [44, 57], filter design [43, 49], and so on. *An optimal solution could mean further improvement of the signal.* Our advance in [38] is made possible by formulating the structured low rank approximation as constrained optimization problems which are then solved by a high-power large-scale optimization package LANCELOT available from [39]. In this research, the PI plans to extend the results in [38] to include general structures. The PI also plans to compare the performance of different formulations and to experiment with other optimization routines in [39].

Inverse Testing Problem. It is perhaps valid to state that eigenvalues and singular values are two of the most distinguished characteristics in any given general square matrix. Over the years, one of the most fruitful developments in numerical linear algebra, which serves as a computational platform for a variety of application problems, is that of efficient and stable algorithms for the computation of eigenvalues and singular values of a given matrix. On the other hand, it has long been desired to construct matrices that has prescribed eigenvalues and singular values. *Such a construction, if possible, would enable us to create matrices with desired spectral features for the purpose of algorithm testing.* Although Wyle and Horn have laid down the theoretical foundation of necessary and sufficient conditions five decades ago [53, 68], only recently did the numerical construction of such matrices become possible using a fast recursive algorithm developed by the PI [34]. Numerical experiments have shown that this technique can create interesting complex-valued matrices that have the same spectral properties as some of the most challenging benchmark matrices, such as the Rosser matrix and the Wilkinson matrix. In this proposal, the PI plans to modify the technique so that real-valued matrices can be constructed. The PI has found a critical clue from the paper by Thompson [65, Lemma 1] and is working on the development of a powerful tool that can generate real-valued matrices with desired eigenvalues and singular values for algorithm testing.

Inverse Eigenvalue Problem. An inverse eigenvalue problem concerns the reconstruction of a matrix from prescribed spectral data. The spectral data involved may consist of the complete or only partial information of eigenvalues or eigenvectors. The objective of an inverse

eigenvalue problem is to construct a matrix that maintains a certain specific structure as well as that given spectral property [8]. One of the basic problems in classical vibration theory, for example, is to determine the natural frequencies and normal modes of the vibrating body. *The inverse eigenvalue problem then is to construct a model of a given type, e.g., a mass-spring system, a string, and so on, with prescribed spectral nature.* Thus inverse eigenvalue problems have practical value to applied mechanics, non-destructive testing, and structure design. Studies on inverse eigenvalue problems have been intensive and diverse, ranging from engineering application to algebraic theorization. Recently the PI has gathered 204 references and reviewed a collection of 37 different types of inverse eigenvalue problems in the article [28]. Applications of inverse eigenvalue problems to control design, seismic tomography, and mechanical system were demonstrated in [28]. Specifically, in [28] the PI has attempted to identify and classify the general characteristics of each problems. Up-to-then developments in both the theoretic and the algorithmic aspects were also discussed. Ever since the paper [28] was published, the PI has had the opportunity to come across lots of additional research results on the very subject of inverse eigenvalue problems in the literature. This scope of activities and range of applications strongly signifies the importance of inverse eigenvalue problems. Despite the many efforts found in the literature, however, only a handful of the problems have been completely understood or solved. Many of the open problems pointed out by the PI in [28], including issues of solvability, numerical algorithms, and sensitivity analysis, remain unsettled. The PI has many years' experiences in this subject of inverse eigenvalues problems and intends to continue the investigation in this research.

Matrix Completion Problem. It has been observed that the main diagonal entries and the eigenvalues of any Hermitian matrix enjoy an interesting relationship. This relationship is completely characterized by what is now known as the Schur-Horn theorem [54]. Similarly the Sing-Thompson theorem [66] characterizes the relationship between the diagonal entries and the singular values of an arbitrary matrix. These theoretical results are special cases of the larger class of matrix completion problems where partially described matrices are to be completed with prescribed eigenvalue or singular values. *Matrix completion problems often arise when the underlying physical system is not free standing so that a portion of its data matrix must have fixed entries to satisfy certain physical constraints.* In [24, 71] two different numerical methods, one using the flow approach and the other divide-and-conquer, were developed to construct matrices with fixed diagonal entries and eigenvalues. In [29] the PI also developed a recursive method for constructing matrices with fixed diagonal entries and singular values. In practice, however, the physical dependence might not leave fixed data along the diagonal at all. The PI wants to consider the generalization of results in [24, 29] when other part of the data matrix are fixed.

Matrix reconstruction problems not only are of theoretical interest, but in every case discussed above there are important physical, medical, and engineering applications involved. The techniques develop through this research effort are expected to have substantial impact on the progress of scientific and engineering fields.