MA 580 HOMEWORK 1 Name: _____ Due 10/31

1. (The Cholesky Decomposition Theorem) Let A be a real, symmetric and positive definite matrix. Show that A can be decomposed in exactly one way into the product

$$A = LL^T$$

where L is lower triangular and has positive entries on the main diagonal. (This is a special case of the LU decomposition of a general matrix.)

2. By a "flop", we roughly mean the effort of doing one floating-point addition, one floating-point multiplication, and a little subscripting. Usually the overhead of a numerical algorithm is assessed by the number of flops involved in the computation. Estimate the overhead needed to solve $Ax = b$ by the Gaussian method.

3. Let A be a nonsingular matrix and B be a singular matrix. Let $\|A\|$ denote an induced matrix norm. Use Banach's lemma to show that

$$\|A - B\| \geq \|A^{-1}\|^{-1}.$$

That is, the distance from A to its nearest singular matrix is at least $\|A^{-1}\|^{-1}$.

4. Propose an efficient way to compute the inverse of a lower triangular matrix and justify your proposal.

5. Let H_n denote the $n \times n$ Hilber matrix.

   (a) Use MATLAB commands **inv(hilb(n))** and **invhilb(n)** to compare the difference between a computed H_n^{-1} and the exact H_n^{-1}.

   (b) Compare the average error introduced per flop between a computed H_n^{-1} and the exact H_n^{-1}.