MA/CSC 580 PROJECT 2 Name: _____ Due October 5, 2000

Consider the Dirichlet boundary value problem (Poisson's Equation)

$$-u_{xx} - u_{yy} = f(x,y) \quad (1)$$

$$u(x,y) = 0 \text{ for } (x,y) \in \partial\Omega \quad (2)$$

over the domain Ω below.

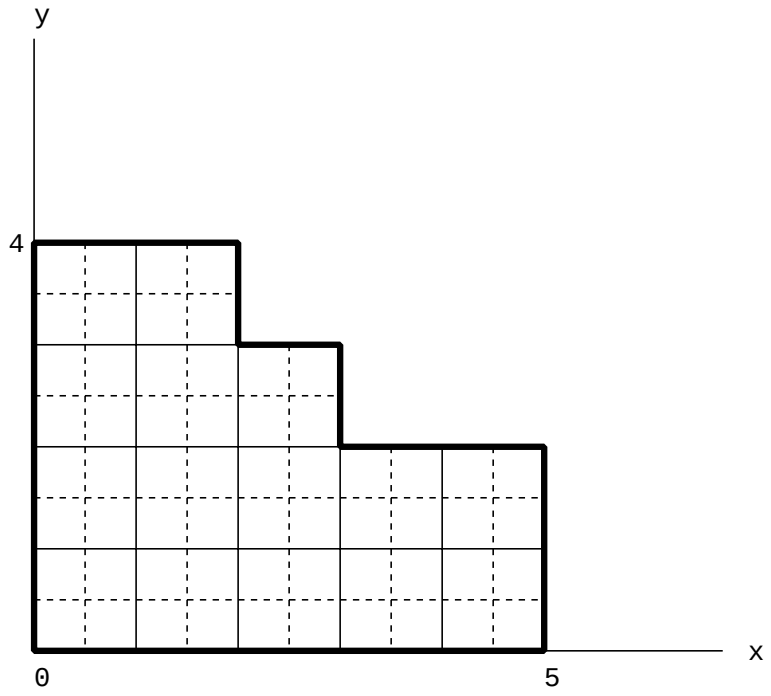


Figure 1: Domain Ω.

1. Derive the associated system of equations $Ax = b$, using the discretization that the differential operator $-u_{xx} - u_{yy}$ at the point $(x_i, y_j) \in \Omega$ is replaced by

$$\frac{4u_{ij} - u_{i-1,j} - u_{i+1,j} - u_{i,j-1} - u_{i,j+1}}{h^2}$$

   where h is the grid size in either x and y direction.

   (a) Take $h = 1$. (The real value of h is immaterial. Imagine that things have been scaled.)

   (b) Redo the above problem by halving the grid size.

2. Select some non-trivial $f(x,y)$ for the right-hand side. Solve the corresponding linear systems. Represent your (discrete) solutions by 3-D graphs.