MA/CSC 580 PROJECT 3 Name: _____ Due October 5, 2000

The purpose of the project is to apply the PCG method to a simulated image restoration problem with a spatially invariant point spread function. To avoid the technical details, follow the steps below carefully and fill in the details as you go through them.

1. Use the following Matlab commands

```
f=zeros(64);
f(21:44,20:30) = 1;
f(16:49,40:45) = 2;
f(11:55,50:55) = 3;
f(30:40,50:55) = 2.2;
mesh(f);
```

   to simulate a 64×64 image of high-rise office block.

2. Through lens with finite aperture, the original image will be aberrated by a point spread function (PSF) h with finite support. The image g perceived through the lens can be modeled by the convolution equation

$$g = h * f. \tag{1}$$

   More precisely, the observed image is given by

$$g(i,j) = \sum_{m,n \in S_h} h(m,n) f(i-m, j-n), \tag{2}$$

   where S_h stands for finite extent region (support) where $h(m,n) \neq 0$.

3. Consider the spatially invariant discretized PSF matrix H defined by

$$h_{i-j,k-\ell} = \begin{cases} e^{-0.1((i-j)^2+(k-\ell)^2)}, & \text{if } -8 \leq i-j, k-\ell \leq 8, \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

   In the image processing community this H is known as a Gaussian PSF. Use the following Matlab commands

```
[X,Y] = meshgrid(-8:8,-8:8);
Z = exp(-0.1*(X.^2+Y.^2));
mesh(-8:8,-8:8,Z);
```

   to gain an idea about the shape of the PSF h defined above.

4. Suppose that an $M \times N$ 2-D image F is concatenated by rows into an $MN \times 1$ vector F. Show that the convolution equation (1 can be rewritten as a matrix equation

$$\mathrm{g} = H\mathrm{f} \tag{4}$$

where H is a $MN \times MN$ matrix that has Block-Toeplitz with Toeplitz-Block structure. That is, H is of the form

$$H = \begin{bmatrix} H_0 & H_{-1} & H_{-2} & & 0 \\ H_1 & H_0 & H_{-1} & & \\ H_2 & H_1 & H_0 & \dots & \\ & & \vdots & \ddots & \vdots \\ 0 & & & \dots & H_0 \end{bmatrix} \tag{5}$$

where

$$H_j = \begin{bmatrix} h(j,0) & h(j,-1) & h(j,-2) & & 0 \\ h(j,1) & h(j,0) & h(j,-1) & & \\ h(j,1) & h(j,1) & h(j,0) & \dots & \\ & & \vdots & \ddots & \vdots \\ 0 & & & \dots & h(j,0) \end{bmatrix}. \tag{6}$$

5. Apply the Gaussian PSF (3) to the high-rise office block image. Plot the blurred image. (Note that the resulting H will be of size 4096×4096.)

6. Restore the image f using the CG method.

7. Reconsider the problem when the image f is of one dimension. How does the resulting blurring matrix H should look like?

8. Verify that the PCG algorithm, **conjgrad**, in the book on page 157 is valid.

9. Restore the 1-D image using the PCG method whereas the preconditioner is the circulant matrix C whose entries are the average of the diagonals of the blurring matrix H.

References

[1] R. H. Chan, J. G. Nagy, and R. J. Plemmons, FFT-based preconditioners for Toeplitz-block least squares problems, SIAM J. Numer. Anal., 30(1993), 1740-1768.

[2] T. Chan, An optimal circulant preconditioner for Toeplitz systems, SIAM J. Sci. Statist. Comput., 9(1988), 766-771.