The numerical integration command commonly used in Maple

$$evalf(int(f, x = a..b))$$

is quite different from the Gaussian quadrature we discussed in class. The default numerical method used in Maple is called Clenshaw-Curtis quadrature that explicitly and directly defines the abscissas in the interval $[-1, 1]$ to be these points:

$$x_k := \cos\left(\frac{k-1}{n-1}\pi\right), \quad k = 1, \ldots, n.$$

Such a scheme, as we pointed out in class, has less degree of precision than the Gaussian quadrature. This project asks you to experiment with Gaussian quadrature.

1. Generate the Gaussian abscissas and weights.

   (a) Use the available library **orthopoly** in Maple to find out the first six orthogonal polynomials, $P_k(x)$, $k = 0, 1, \ldots, 5$.

   (b) Plot these orthogonal polynomials on the same frame.

   (c) Use Maple or Matlab to find out the *roots* of these polynomials up to the 16th significant digits. (Note that these roots are the abscissas in the corresponding Gaussian quadrature.)

   (d) Recall that

   $$\alpha_i = \int_{-1}^{1} \ell_i(x)\, dx$$

   where $\ell_i(x)$ is simply the Lagrange basis polynomial

   $$\ell_i(x) = \prod_{j=1, j\neq i}^{n} \frac{x - x_j}{x_i - x_j},$$

   use the Gaussian abscissas found above to calculate the corresponding Gaussian weights α_i.

2. Apply the 5-point Gaussian quadrature to evaluate the following double integral:

   $$\int_0^{\pi/4} \int_0^{\sin x} \frac{2y \sin x + (\cos x)^2}{\sqrt{1 - y^2}}\, dy\, dx$$

3. Compare your numerical answer with that obtained from Maple or your calculator calculation.