MA 427 FINAL EXAM Name: _____

December 10, 2003 SS #: _____

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1. (3 pts) Explain the following terms:
   (a) Round-off errors,
   (b) Truncation errors, and
   (c) Machine accuracy.
2. (9 pts) Tabulate all numbers in fractional form that can be exactly represented in a system with base $\beta = 3$, mantissa length $t = 2$, exponent length $q = 1$, and exponent shift by 2?
3. (5 pts each) Given a set of points $\{(x_i, f_i)\}_{i=0}^{n}$,
   (a) Describe the Lagrange formula that interpolates these points.
   (b) Describe the Newton formula that interpolate these points.
   (c) Discuss the pros and cons of each formula.
4. (5 pts each) Concerning the numerical integration:
   (a) Describe the composite trapezoidal rule and derive its error formula.
   (b) What is the fundamental difference between the Newton-Cotes quadrature and the Gaussian quadrature?
   (c) Briefly explain how does the theory of orthogonal polynomials help to derive the Gaussian quadrature?
   (d) Describe how an integral over a general interval can be handled if we know the Gaussian quadrature over the interval $[-1, 1]$.
5. (5 pts) What is the limitation on numerical differentiation?
6. (5 pts each) Concerning the Runge-Kutta methods:
   (a) Describe the format of a general R-stage Runge-Kutta method.
   (b) Discuss the stability properties of a general R-stage RK method.
   (c) Calculate the growth factor of a 3-stage explicit RK method.
7. (5 pts each) Concerning the multi-step methods:
   (a) Describe the format of a general linear p-step method.
   (b) Compare the differences between the RK method and the multi-step method.
   (c) What is the Dalquist barrier theorem?
   (d) Describe the general idea in a predictor-corrector method
8. (10 pts) What is the basic scheme in a Newton method? Describe the geometric meaning of a one-dimensional Newton iteration.