This homework (therefore, to be done by individuals) asking you to set up some theoretical schemes that transform an abstract integral equation into an algebraic equation. Consider numerical methods for solving the Fredholm integral equation of the second kind, i.e.,

$$\varphi(x) - \int_a^b K(x,y)\varphi(y)dy = f(x), \quad x \in [a,b] \tag{1}$$

with continuous kernel K and $\varphi \in C[a,b]$.

1. (The Nyström method) Suppose that $\varphi(x)$ is represented by a set of discrete data $\varphi_n := [\varphi(x_0), \ldots, \varphi(x_n)]^T$ at quadrature points, i.e., abscissas, $x_0, \ldots, x_n \in [a,b]$. Describe how the solution to the integral equation (1) is approximated by the solution of a linear system

$$\varphi_n - A_n\varphi_n = f_n. \tag{2}$$

   Be specific about entries in the matrix A_n.

2. (The collocation method) Suppose that $\varphi(x)$ is approximated by the Lagrange interpolation polynomial

$$\varphi_n(x) = \sum_{k=0}^{n} \gamma_k \ell_k(x) \tag{3}$$

   where $\ell_k(x)$, $k = 0, \ldots, n$, are the Lagrange factors

$$\ell_k(x) = \prod_{i=1, i\neq k}^{n} \frac{x - x_i}{x_k - x_i} \tag{4}$$

   at a predesignated set of $n+1$ points $x_0, \ldots, x_n \in [a,b]$ (called the collocation points.) It is desired that the approximate solution $\varphi_n(x)$ satisfies (1) at all points $x_0, \ldots, x_n \in [a,b]$. Describe how the values γ_k should be determined from a linear system.