MA/CSC 428 PROJECT 1 Name: _____ Due February 13, 2001

Consider the Dirichlet boundary value problem (Poisson's Equation)

$$-u_{xx} - u_{yy} = f(x, y, t) \quad (1)$$
$$u(x, y) = 0 \text{ for } (x, y) \in \partial\Omega \quad (2)$$

over the domain Ω below.

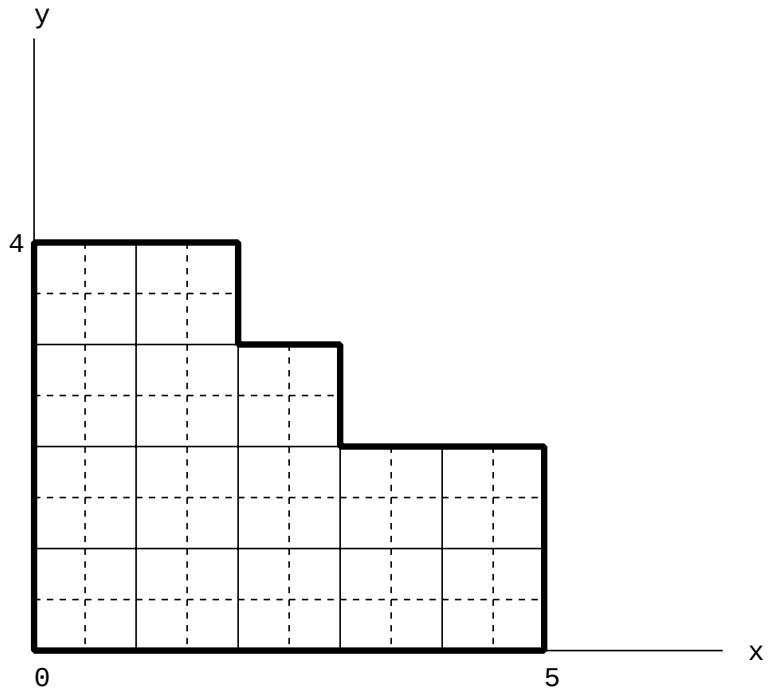


Figure 1: Domain Ω.

1. For each fixed t, derive the associated system of equations $Au(t) = b(t)$, using the discretization that the differential operator $-u_{xx} - u_{yy}$ at the point $(x_i, y_j) \in \Omega$ is replaced by
$$\frac{4u_{ij} - u_{i-1,j} - u_{i+1,j} - u_{i,j-1} - u_{i,j+1}}{h^2}$$
where h is the grid size in either x and y direction.

   (a) Begin with $h = 1$ to see how the system should be set up.

   (b) After you have learned how to set up the linear system, select some non-trivial $f(x, y, t)$ for the right-hand side of the differential system. Solve the above problem by using much smaller grid size.

2. Represent your (discrete) solutions by 3-D graphs. For each t, create a frame so that later you can play back the movie. The following is an example for assembling an array of movie frames:

```
for j=1:n
   plot_command
   M(j) = getframe;
end
movie(M)
```

whereas the "plot_command" should something like "mesh(u)" if the vector solution u obtained from the linear system is mapped back to the layout of grids over Ω.