Consider the matrix

$$A = \begin{bmatrix} 4.0988 & -0.0641 & -1.9944 & -0.4153 & 0.1109 \\ -1.2161 & 4.3421 & 11.3103 & 2.7057 & 3.3299 \\ 0.7007 & -0.4686 & -2.0851 & -1.1957 & -0.6582 \\ 1.9529 & -1.0422 & -11.7624 & 0.2374 & -1.7036 \\ -1.9899 & 1.7796 & 7.9713 & 1.4742 & 4.4067 \end{bmatrix}.$$

1. Find its eigenvalues and eigenvectors.

2. Write a program that performs the zero-shift QR algorithm. Repeat the iteration 40 times. Report what you have observed by plotting the diagonal entries, the subdiagonal entries, and the $(1, 5)$ entry against the number of iteration.

3. Write a program that performs the Rayleigh quotient iteration. Repeat the iteration 40 times. Report what you have observed in the same way as above. Also, plot the Rayleigh quotients.

4. Write a program that does the shift QR algorithm with constant shifts 4 and 3, respectively. Report what you have observed in the same way as above.