Prove that the Chebyshev polynomials given by

$$T_k(x) := \cos(k \arccos x), \quad , k = 0, 1, \ldots$$

are indeed polynomials satisfying the recurrence relationship

$$\begin{aligned} T_0(x) &\equiv 1, \\ T_1(x) &= x, \\ T_{k+1}(x) &= 2xT_k(x) - T_{k-1}(x) \end{aligned}$$