Assume $n = 2m$ and define $\omega_n := e^{-2\pi i/n}$. Let W_n denote the inverse of the Fourier matrix F_n, i.e.,

$$W_n = [\omega_n^{pq}], \quad p, q = 1, \ldots, n.$$

Define further the diagonal matrix

$$\Omega_m = \mathrm{diag}(1, \omega_n, \ldots, \omega_n^{m-1}),$$

and the permutation matrix

$$\Pi_n = I(:, [0 : 2 : n-1, 1 : 2 : n-1]).$$

Prove that

$$W_n \Pi_n = \begin{bmatrix} W_m & \Omega_m W_m \\ W_m & -\Omega_m W_m \end{bmatrix}.$$

(Hint: Consider the four blocks respectively by taking into account that $\omega_n^2 = \omega_m$ and $\omega_n^m = -1$.)