1. Derive step by step *qualitatively* a basic cubic B-spline $B_{i,4}(x)$ over the knot sequence $\{t_i, t_{i+1}, t_{i+2}, t_{i+3}, t_{i+4}\}$, assuming all these breakpoints are distinct.

2. The cubic B-spline is made of four pieces of cubic polynomials. Let these polynomials be called b_2, b_1, b_0, and b_{-1}, starting from the leftmost piece. Show that if each piece is defined over the interval $[0, 1]$, i.e, suppose we re-scale the interval $[t_k, t_{k+1}]$ into $[0, 1]$, then the polynomials are given, respectively, by

$$b_{-1} = \frac{(1-u)^3}{6} \tag{1}$$

$$b_0 = \frac{u^3}{2} - u^2 + \frac{2}{3} \tag{2}$$

$$b_1 = -\frac{u^3}{2} + \frac{u^2}{2} + \frac{u}{2} + \frac{1}{6} \tag{3}$$

$$b_2 = \frac{u^3}{6}, \tag{4}$$

for $0 \leq u \leq 1$. (Note: For this reason, many CAD users compute without the knowledge of what we were doing in class by saying that the cubic spline $S_i(u)$ over the interval $[t_i, t_{i+1}]$ is given by

$$S_i(u) = \sum_{k=-1}^{2} b_k(u) f(t_{i+k}) \tag{5}$$

with $0 \leq u \leq 1$. You should realize that there are four nontrivial B-splines over the interval $[t_i, t_{i+1}]$. Each of these four B-splines has a section active over $[t_i, t_{i+1}]$. The b_k, being parameterized by u over $[0, 1]$, can universally represent that active section. This explains why (5) is so popularly used by CAD engineers.)