MA 141 TEST # 4 Name: _____
November 29 SS #: _____
Show All Work Row #: _____

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1. (10 pts each) Find the limit.
   (a) $\lim_{x\to 0}(1-2x)^{1/x}$.
   (b) $\lim_{x\to\infty}\frac{\ln(\ln x)}{\ln x}$.
   (c) $\lim_{x\to 1}(\frac{1}{\ln x}-\frac{1}{x-1}$.
2. (10 pts) A paper cup has the shape of a cone with height 10 *cm* and radius 3 *cm* (at the top). If water is poured into the cup at a rate of 2 cm^3/s. How fast is the water level rising when the water is 5 *cm* deep?
3. (10 pts) Find the point on the hyperbola $xy = 8$ that is closest to the point $(3, 0)$.
4. (15 pts each) For each of the following functions, sketch the corresponding graph by identifying all the important aspects of the curve: intervals of increase or decrease, critical points, extreme values, inflection points, intervals of concavity, and so on.
   (a) $f(x) = \frac{x^2-1}{x^3}$.
   (b) $g(x) = 4x^4 - 7x^2 + 4x + 6$.
5. (10 pts) Find the absolute maximum and absolute minimum values of $f(x) = x^2 + \frac{2}{x}$ over the interval $[\frac{1}{2}, 2]$.
6. (10 pts) Let v_1 be the velocity of light in air and v_2 be the velocity of light in water. According to Fermat's Principle, a ray of light will travel from a point A in the air to a point B in the water by a path that minimized the time taken. Show that

$$\frac{\sin\theta_1}{sin\theta_2} = \frac{v_1}{v_2}$$

   where θ_1 is the angle of incidence and θ_2 is the angle of refraction. (*Hint: See the drawing on page 318 in the book*).