MA 242 FINAL EXAM Name: ______________
May 5, 1997 SS #: ______________
Show All Work Seat #: ______________

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1. (10 pts) Find an equation of the plane that contains the points $(1, 0, -1)$ and $(2, 1, 3)$ and is orthogonal to $2x - y + 3z = 6$. $\mathbf{7x + 5y - 3z = 10}$

2. (10 pts) Find parametric equations of the line that passes through the point $(1, -1, 1)$, is orthogonal to the line $x = \frac{1}{3}t, y = \frac{1}{2}t, z = t$, and is parallel to the plane $x + y - z = 0$. $\mathbf{x = 1 - \frac{3}{2}t,\ y = -1 + \frac{4}{3}t,\ z = 1 - \frac{1}{6}t}$

3. (10 pts) Decompose the vector $\vec{b} = [2, 3, -1]$ into a sum $\vec{b}_1 + \vec{b}_2$, where $\vec{b}_1$ is parallel to $\vec{a} = [0, 4, 2]$ and $\vec{b}_2$ is orthogonal to $\vec{a}$. $\vec{\mathbf{b}}_\mathbf{1} = [\mathbf{0}, \mathbf{2}, \mathbf{1}],\ \vec{\mathbf{b}}_\mathbf{2} = [\mathbf{2}, \mathbf{1}, -\mathbf{2}]$

4. (10 pts) Find the equation of the tangent plane to the graph of $f(x, y) = \ln(x^2 + y^2)$ at the point $P(0, 1, 0)$. $\mathbf{2y - z = 2}$

5. (5 pts each) Suppose a shot is launched from $6.5ft$ above the ground at an angle of $\pi/4$ radians, and is landed with a horizontal distance of $69.75ft$. Find
   (a) The initial speed. $\mathbf{s = 69.75 * \sqrt(2)/\frac{\sqrt{76.25}}{4} \approx 45.18551921}$
   (b) The maximum height. $\mathbf{t_{height} = s/(32\sqrt(2) \approx 0.9984683447,\ height \approx 22.45102456}$
   (c) The time of flight. $\mathbf{t_{flight} = \frac{\sqrt{76.25}}{4} \approx 2.183031150}$ *(Hint: The equation of motion is $x(t) = x'(0)t + x(0)$, $y(t) = -16t^2 + y'(0)t + y(0)$.)*

6. (10 pts) The curve $y = \cos x$ from $x = 0$ to $x = \frac{\pi}{2}$ is revolved about the x-axis. Derive the equation of the surface. $\mathbf{y^2 + z^2 = \cos x}$

7. (10 pts each) Find the derivatives: **do it by yourself**
   (a) $\frac{\partial f}{\partial u}$ where $f(x, y) = e^{xy}(\cos xy + \sin xy)$, $x(u, v) = e^{u+v}$ and $y(u, v) = uv$.
   (b) $\frac{df}{dt}$ where $f(x, y, z) = e^z \cos xy$, $x(t) = \sin t$, $y(t) = t^2 + 1$ and $z(t) = e^{t^2}$.

8. (10 pts) For the function $f(x, y) = 6x^2 - 2x^3 + 3y^2 + 6xy$, find all critical points and determine any relative extrema or saddle points. $\mathbf{(0, 0), localmax;\ (1, -1), saddle}$

9. (10 pts each) Set up, but do not evaluate, the integral for the following tasks.
   (a) Find the volume of the solid that is inside the cylinder $r = \sin\theta$, above the paraboloid $z = x^2 + y^2 - 4$, and below the xy-plane. $\int_\mathbf{0}^\pi \int_\mathbf{0}^{\sin\theta} \int_{\mathbf{4-r^2}}^\mathbf{0} \mathbf{rdzdrd}\theta$
   (b) Find the volume of the solid bounded by $z = \sqrt{x^2 + y^2}$, $z = \sqrt{3x^2 + 3y^2}$ and $x^2 + y^2 + z^2 = 9$. $\int_\mathbf{0}^{\mathbf{2}\pi} \int_{\pi/\mathbf{4}}^{\pi/\mathbf{3}}, \int_\mathbf{0}^\mathbf{3} \rho \sin^\mathbf{2} \phi \mathbf{d}\rho \mathbf{d}\phi \mathbf{d}\theta$
   (c) Rewrite the integral $\int_0^1 \int_{-\sqrt{1-z}}^{\sqrt{1-z}} \int_0^{\sqrt{1-y^2-z}} y\, dx\, dy\, dz$ in the order $dz\, dx\, dy$. $\int_\mathbf{0}^\mathbf{1} \int_\mathbf{0}^{\sqrt{\mathbf{1-y^2}}} \int_\mathbf{0}^{\mathbf{1-x^2-y^2}} \mathbf{ydzdxdy}$
   (d) The work done by the force $\vec{F}(x, y, z) = y \sin x\vec{\imath} + ye^x\vec{\jmath}$ along the cure $\vec{r}(t) = t\vec{\imath} + \cos t\vec{\jmath} + \vec{k}$, for $0 \le t \le \pi/2$.

10. (10 pts each) For the integral

$$\int_{-1}^{2} \int_0^1 \int_0^{\sqrt{1-x^2}} e^{x^2+y^2}\, dy\, dx\, dz$$

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   (a) Sketch the solid of integration. **a section of a cylinder**
   (b) Evaluate the integral. **change to cylindrical,** $\mathbf{\frac{3\pi}{4}(e - 1)}$

11. (10 pts) Find the volume of the solid with boundaries $z = \sqrt{x^2 + y^2}$, $x^2 + y^2 = 4$, $x^2 + y^2 = 9$, and $z = 0$.

12. (10 pts) Name and sketch the quadratic surface $x^2 - 4y^2 - 9z^2 = 36$.

13. (15 pts) Show that the vector field $\vec{F}(x, y, z) = 2xy\vec{\imath} + (x^2 + \sin z)\vec{\jmath} + (y \cos z + 2)\vec{k}$ is conservative, and find its potential function.

14. (10 pts) Use Green's Theorem to find the area enclosed by the curve $x = \cos t, y = \sin t \cos t$, for $-\pi \le t \le \pi$.