MA 242 TEST # 2 Name: ____________________

February 21, 1997 SS #: ____________________

Show All Work Row #: ____________________

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1. (8 pts each) Find the derivatives. (No need to simplify your answers.)
   (a) $\frac{\partial g}{\partial x}$ where $g(x,y) = \ln(ye^{xy})$. $\mathbf{y}$
   (b) $\frac{\partial z}{\partial v}$ where $z = \sin(x+y^2)$, $x = u^2+v^2$ and $y = uv$. $\cos(\mathbf{u^2+v^2+u^2v^2})\,(\mathbf{2\,v+2\,u^2v})$
   (c) $\frac{dz}{dt}$ where $z = (x+y)e^y$, $x = \ln t$ and $y = \frac{1}{t^2}$. $\left(\mathbf{t^{-1}} - \frac{\mathbf{2}}{\mathbf{t^3}}\right)\mathbf{e^{t^{-2}}} - \left(\mathbf{2}\,\ln(\mathbf{t}) + \frac{\mathbf{2}}{\mathbf{t^2}}\right)\mathbf{e^{t^{-2}}t^{-3}}$
   (d) $\frac{\partial^2 z}{\partial x \partial y}$ where $z = \tan^{-1}(\frac{x}{y})$. $-\mathbf{y^{-2}}\left(\mathbf{1}+\frac{\mathbf{x^2}}{\mathbf{y^2}}\right)^{-\mathbf{1}} + \mathbf{2\,x^2y^{-4}}\left(\mathbf{1}+\frac{\mathbf{x^2}}{\mathbf{y^2}}\right)^{-\mathbf{2}}$
   (e) $\frac{dy}{dx}$ where $\sin(xy) + xy^2 = 3$. $-\frac{\cos(\mathbf{xy})\mathbf{y}+\mathbf{y^2}}{\cos(\mathbf{xy})\mathbf{x}+\mathbf{2xy}}$

2. Given the function $f(x,y) = x^2 - y^3$,
   (a) (10 pts) Find the directional derivative at the point $(1,2)$ in the direction making an angle $\theta = \frac{2\pi}{3}$ with the x-axis. $\vec{\mathbf{u}} = \langle -\frac{\mathbf{1}}{\mathbf{2}}, \frac{\sqrt{\mathbf{3}}}{\mathbf{2}}\rangle,\ \ \mathbf{D_{\vec{u}}f(1,2) = -1 - 2\sqrt{3}}$
   (b) (5 pts) Find the largest directional derivative. $\sqrt{\mathbf{20}}$

3. Given two planes $2x - 3y + 5z = 2$ and $4x + y - 3z = 7$,
   (a) (5 pts) Explain why these two planes intersect. **Planes are not parallel because their normals are not.**
   (b) (5 pts) Find a vector parallel to the intersection of the two planes. $\langle \mathbf{4, 26, 14} \rangle$
   (c) (5 pts) Find the equation of the plane that passes through the point $(1,2,-1)$ and is perpendicular to the line of the intersection of the planes. $\mathbf{2(x-1) + 13(y-2) + 7(z+1) = 0}$

4. Given the points $P = (0,1,0), Q = (-1,1,2), R = (2,1,-1)$, find
   (a) (10 pts) The area of the triangle PQR. $\frac{\sqrt{\mathbf{9}}}{\mathbf{2}}$
   (b) (10 pts) The equation for a plane that contains P, Q, and R. $\mathbf{y - 1 = 0}$

5. (10 pts) Find an equation of the tangent plane to the surface $z = \frac{1}{2}(x^2 + 4y^2)$ at the point $(2,1,4)$. $\mathbf{z = 4 + 2(x-2) + 4(y-1)}$