MA 242 | TEST # 3 | Name: ____________
March 21, 1997 | | SS #: ____________
Show All Work | | Row #: ____________

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1. (10 pts each) For the function $f(x, y) = x^2 + y^4 - y^2 - 2xy$,
   (a) Find all the critical points. $\mathbf{(0, 0), (1, 1), (-1, -1)}$.
   (b) Classify properties of these critical points.
   $\mathbf{(0, 0)}$ is a saddle point; the other two are maximizers.

2. (10 pts each) Concerning the objective function $f(x, y) = x^2 + y$ with x, y satisfying the constraint $x^2 - y^2 = 1$,
   (a) Use Lagrange multipliers to find all the critical points.
   $(\frac{\sqrt{5}}{2}, -\frac{1}{2}), (-\frac{\sqrt{5}}{2}, -\frac{1}{2})$ with $\lambda = \mathbf{1}$.
   (b) Use level curves of f to identify the maximizer(s) and the minimizer(s) of f on the constraint.
   As levels are increasing when curves moves upward, these two points are minimizers.

3. Consider the three-sided pyramid whose base is the xy-plane and whose three sides are the vertical plans $y = 0$, and $y - x = 4$ and the slanted plane $2x + y + z = 4$.
   (a) (10 pts) Sketch the solid. Identify the edges. **Done in class. Problem #21 on page 243.**
   (b) (10 pts) Set up the integral for the volume of the pyramid.
   $\int_0^4 \int_{y-4}^{(4-y)/2} 4 - 2x - y \, dx \, dy$
   (c) (5 pts) Evaluate the volume. **48**

4. Perform the following tasks as required:
   (a) (10 pts) Evaluate $\int_0^3 \int_{y^2}^9 y \sin x^2 \, dxdy$ by reversing the order of integration.
   $\int_0^3 \int_0^{\sqrt{x}} y \sin x^2 \, dy \, dx = \frac{1 - \cos 81}{4}$.
   (b) (10 pts) Sketch the region of integration for the triple integral

$$\int_0^3 \int_{-\sqrt{9-y^2}}^0 \int_{\sqrt{x^2+y^2}}^3 f(x, y, z) \, dzdydx.$$

5. An ice cream cone con be modeled by the region bounded by the hemisphere $z = \sqrt{8 - x^2 - y^2}$ and the cone $z = \sqrt{x^2 + y^2}$.
   (a) (10 pts) Set up the integral for the volume in polar coordinates.
   $\int_0^{2\pi} \int_0^2 (\sqrt{8 - r^2} - r) \, r \, dr \, d\theta$.
   (b) (5 pts) Find its volume. $\frac{32}{3}(\sqrt{2} - 1)\pi$