MA 242 TEST # 4 Name: _____
April 17, 1997 SS #: _____
Show All Work Row #: _____

Note: For all the integrals to be involved, you may use any coordinate systems you like to do the computation. Partial credits will be given only when you have provided enough information.

1. A solid is bounded above by $z = 12 - 2x^2 - 2y^2$ and below by $z = x^2 + y^2$

   (a) (5 pts) Sketch the solid.

   (b) (10 pts) Set up the integral for the volume of this solid. $\int_{\mathbf{0}}^{\mathbf{2\pi}} \int_{\mathbf{0}}^{\mathbf{2}} \int_{\mathbf{r^2}}^{\mathbf{12-2r^2}} \mathbf{r}\,\mathbf{dzdrd}\theta$

   (c) (10 pts) Evaluate the integral.

2. (10 pts) Set up, but do not evaluate, the integral for the volume that remains after a cylindrical hole of radius R is bored through a sphere of radius a, where $0 < R < a$, passing through the center of the sphere along the pole. $\int_{\mathbf{0}}^{\mathbf{2\pi}} \int_{\mathbf{R}}^{\mathbf{a}} \int_{\mathbf{-\sqrt{a^2-r^2}}}^{\mathbf{\sqrt{a^2-r^2}}} \mathbf{r}\,\mathbf{dzdrd}\theta$

3. A solid is bounded by $z = x^2 + y^2$, $z = 1$ and $z = 4$.

   (a) (10 pts) Set up the integral for the volume. $\int_{\mathbf{1}}^{\mathbf{4}} \int_{\mathbf{0}}^{\mathbf{2\pi}} \int_{\mathbf{0}}^{\mathbf{\sqrt{z}}} \mathbf{r}\,\mathbf{drd}\theta\mathbf{dz}$, **(Watch out the order of integration)**

   (b) (5 pts bonus) Evaluate the integral.

4. Two particles are traveling through space. At time t the first particle is at the point $(-1+t, 4-t, -1+2t)$ and the second is at $(-7+2t, -6+2t, -1+t)$. **Done in class.**

   (a) (10 pts) Will the two particles collide, and if so when and where?

   (b) (10 pts) Will the paths of the two particles cross, and if so where?

5. Find a parameterization for each of the following geometric objects:

   (a) (10 pts) The spherical ring $x^2 + y^2 + z^2 = 1$, with $-\frac{1}{2} \le x \le -\frac{1}{2}$. (*Hint: The restriction on x should also limits the range of your parameters.*)

   $\mathbf{x = p}$, $\mathbf{y = \sqrt{1-p^2}\cos\theta}$, $\mathbf{z = \sqrt{1-p^2}\sin\theta}$, $\mathbf{-\frac{1}{2} \le p \le \frac{1}{2}, 0 \le \theta 2\pi}$ **(Many other ways.)**

   (b) (10 pts) The plane $x + 2y - z = 4$.

   Many ways. One solution is $\mathbf{x = 4 + 4p + 4q, y = -2p, z = 4q}$.

6. (10 pts) Sketch the surface parameterized by

$$\begin{aligned} x &= r\cos\theta, \quad 0 \le r \le 5 \\ y &= r\sin\theta, \quad 0 \le \theta \le 2\pi \\ z &= 7. \end{aligned}$$

This is a disk of radius 5 **in the plane** $z = 7$.