MA 241 FINAL EXAM Name: _____
June 22, 1999 SS #: _____
Show All Work Seat #: _____

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1. (8 pts each) Find the derivative $\frac{dy}{dx}$:

$$(a)\ y = \sqrt{\arctan\sqrt{x}} \quad (b)\ y = (\cos x)^x \quad (c)\ x^y + y^x = 1$$

2. (8 pts each) Evaluate the integral:

$$(a)\ \int_{1/4}^{1/2} \frac{dx}{\sqrt{x-x^2}} \quad (b)\ \int x^3 \ln x^2\, dx \quad (c)\ \int \sin 2x \cos 3x\, dx \quad (d)\ \int \frac{x^2}{\sqrt{1+x^2}}\, dx \quad (e)\ \int \frac{x^3}{\sqrt{4x^2-9}}\, dx$$
$$(f)\ \int \frac{x^3}{\sqrt{4-9x^2}}\, dx \quad (g)\ \int \arctan x\, dx \quad (h)\ \int \frac{1}{\cos^3 x}\, dx \quad (i)\ \int \frac{5z-13}{6-5z-z^2}\, dz \quad (j)\ \int x\sqrt{1-x}\, dx$$

3. (8 pts) Set up, but do not evaluate, the Simpson's rule for the integral

$$\int_1^2 \frac{sinx}{x}\, dx$$

by using 10 subintervals.

4. (10 pts) Find the volume of the solid whose base is the region in the xy-plane bounded by the curves $y = x^2$ and $y = 8 - x^2$, and whose cross-sections perpendicular to the x-axis are squares with one-side in the xy-plane.

5. (10 pts) The region bounded by the curves $y = x$ and $y = x^2$ is rotated about the line $y = 3$. Compute the volume of the resulting solid.

6. (8 pts each) Find the general solution to the differential equation.

$$(a)\ \frac{dy}{dt} = 2^y \sin^3 t \qquad (b)\ \frac{dw}{dt} = (w-3)(w-7)$$
$$(c)\ e^{-\cos\theta}\frac{dz}{dt} = \sqrt{1-z^2}\sin\theta,\ z(0) = \tfrac{1}{2} \qquad (d)\ y'' + 2y' + 2y = 0,\ y(0) = 2,\ y'(0) = 1$$

7. Concerning the differential system

$$\begin{aligned} \frac{dx}{dt} &= -0.45x + 10.5 \\ \frac{dy}{dt} &= 8.2x - 0.8y - 142 \end{aligned}$$

   (a) (8 pts) Find the nullclines and equilibrium points.

   (b) (8 pts) Analyze the phase plane. Sketch some typical trajectories in the phase plane. Predict what will happen in the long run.

8. (8 pts) Find the first five nonzero terms in the Taylor series of $\arctan x$ about the point $x = 0$.

9. (8 pts) A car moving at speed of v mph achieves $25 + 0.1v$ mpg (miles per gallon) for v between 20 and 60 mph. Suppose your speed as a function of time is given by

$$v = 50\frac{t}{t+1}.$$

   How many gallons of gasoline do you consume between $t = 2$ and $t = 3$?

10. Concerning the differential system

$$\begin{aligned} \frac{dx}{dt} &= y \\ \frac{dy}{dt} &= -x, \end{aligned}$$

   (a) (4 pts) Analyze the phase plane in the region $-2 \le x \le 2$ and $-2 \le y \le 2$.

   (b) (8 pts) Derive a second-order differential equation for $y(t)$, i.e, a equation that involves y and its derivatives, but no x at all. Solve the equation.