MA 241 FINAL EXAM Name: _____
June 22, 1999 SS #: _____
Show All Work Seat #: _____

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1. (8 pts each) Find the derivative $\frac{dy}{dx}$:

$$(a)\ \frac{1}{4\sqrt{\arctan(\sqrt{x})}\sqrt{x}(1+x)} \quad (b)\ (\cos(x))^x\left(\ln(\cos(x)) - \frac{x\sin(x)}{\cos(x)}\right) \quad (c)\ -\frac{y^x \ln y + y x^{y-1}}{x^y \ln x + x y^{x-1}}$$

2. (8 pts each) Evaluate the integral:

$$(a)\ \arcsin(2x-1)\Big|_{1/4}^{1/2} = \frac{\pi}{6} \quad (b)\ \frac{x^4\ln(x^2)}{4} - \frac{x^4}{8} \quad (c)\ -\frac{\cos(5x)}{10} + \frac{\cos(x)}{2}$$
$$(d)\ \frac{x\sqrt{1+x^2}}{2} - \frac{arcsinh(x)}{2} \quad (e)\ \frac{x^2\sqrt{4x^2-9}}{4} - \frac{\left(4x^2-9\right)^{3/2}}{24} \quad (f)\ -\frac{x^2\sqrt{4-9x^2}}{9} - \frac{2\left(4-9x^2\right)^{3/2}}{243}$$
$$(g)\ x\arctan(x) - \frac{\ln(1+x^2)}{2} \quad (h)\ \frac{\sin(x)}{2(\cos(x))^2} + \frac{\ln(\sec(x)+\tan(x))}{2} \quad (i)\ -\frac{43\ln(z+6)}{7} + \frac{8\ln(-1+z)}{7}$$
$$(j)\ -\frac{2(1-x)^{3/2}}{3} + \frac{2(1-x)^{5/2}}{5}$$

3. (8 pts) Simpson's approximation:

$$\int_1^2 \frac{sinx}{x}\,dx$$
$$\approx \frac{.1}{3}\left(\frac{\sin 1}{1} + 4\frac{\sin 1.1}{1.1} + 2\frac{\sin 1.2}{1.2} + 4\frac{\sin 1.3}{1.3} + \ldots + 4\frac{\sin 1.9}{1.9} + \frac{\sin 2}{2}\right)$$

4. (10 pts) Each cross section has area $(8-2x^2)^2$. So volume is

$$\int_{-2}^{2}(8-2x^2)^2\,dx \approx 136.5$$

5. Use the washer method:

$$\int_0^1 2\pi(3-y)(\sqrt{y}-y)\,dy = \frac{13\pi}{15}$$

6. (8 pts each) Find the general solution to the differential equation.

$$(a)\ y(t) = \ln\left(\frac{3}{(\sin(t))^2\cos(t)\ln(2)+2\cos(t)\ln(2)-3C\ \ln(2)}\right)(\ln(2))^{-1} \quad (b)\ w(t) = -\frac{-3+7e^{-4t+4C}}{1-e^{-4t+4C}}$$
$$(c)\ z(\theta) = \sin(-e^{\cos(\theta)} + C) \quad (d)\ y(t) = 2e^{-t}\cos(t) + 3e^{-t}\sin(t)$$

7. (16 pts) Do it by yourself.

8. (8 pts) $\arctan x \approx x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9}$.

9. (8 pts) Gasoline consumed:

$$\int_2^3 \frac{1}{25 + 0.1\frac{50t}{t+1}}\frac{50t}{t+1}\,dt \approx 1.2468\,gals$$

10. (a) (4 pts) DIBY.

    (b) (8 pts) $\frac{d^2y}{dt^2} = -y$.