MA 241 TEST # 3 Name: _____
June 7, 1999 SS #: _____
Show All Work Seat #: _____

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1. A region is bounded by $y = x^3$, $x = 1$ and $y = -1$.

   (a) (5 pts) Sketch the region obtained by rotating the region around the axis $y = -1$.

   (b) (10 pts) Find its volumes.

2. (10 pts) Set up, but do not evaluate, the integral for the volume of the solid generated by rotating the circle $x^2 - 2x + y^2 = 0$ about the y-axis.

3. (15 pts) What is the force of gravitational attraction between a thin uniform rod of mass M and length ℓ and a particle of mass m lying at the bisector of the rod with a distance a from the center of the rod?

4. (10 pts) Show that $y = xe^{kx}$ is a solution of the differential equation $y' = \frac{y}{x}(1+\ln y - \ln x)$.

5. (10 pts each) Find the general solution to the differential equation.

   (a) $\frac{dz}{dt} = z + zt^2$, $z(0) = 5$.

   (b) $\frac{dx}{dt} = tx^2 \sin t^2$, $x(0) = 1$.

6. For the differential equation

$$\frac{dy}{dt} = y^2(1 - y),$$

   (a) (5 pts) Sketch its slope field.

   (b) (10 pts) Solve the equation analytically, but do not try to obtain the expression for $y(t)$ explicitly.

7. (10 pts) Use Euler's method to approximate the solution to the differential equation

$$\begin{aligned} \frac{dB}{dt} &= 0.05B \\ B(0) &= 1000 \end{aligned}$$

   at $t = 1$ by taking step size $h = 0.25$. (*Hint: To go from $t = 0$ to $t = 1$, 4 steps are required.*)