Introduction

- Differentiation is the act or process of differentiating. It also means the development from the one to the many, the simple to the complex, or the homogeneous to the heterogeneous.
  - ◇ Anything that is changing needs to be differentiated.
  - ◇ In mathematics, the *rate of change* of one quantity versus another quantity is measured by the so called *derivative*.
  - ◇ Any equation that involves derivatives is called a *differential equation.*
- The *normal form* of a first-order ordinary differential equation:

$$\frac{dy}{dx} = f(x, y) \tag{0.1}$$

  where
  - ◇ y is the dependent variable, representing, e.g., the location of a spring.
  - ◇ x is the independent variable, representing, e.g., the time.
  - ◇ $f(x, y)$ represents the derivative of y at x.
- We say $y = y(x)$ is a solution to (0.1) if the substitution of $y(x)$ into (0.1) makes its true.
  - ◇ For scalar problems, often we can *see* the solution by plotting the slop field.
  - ◇ Often the solution is *not* unique and, hence, an *initial condition* is needed to uniquely determine a solution.
  - ◇ For general problems, the solution can only be obtained through numerical methods.

Examples

- Consier the DE $y' = x^2 - 2y/x$.
  - ◇ The solution is $y = x^3/5 + Cx^{-2}$.
  - ◇ The slope field can be plotted by Maple commands.

$$with(DEtools):$$
$$DEplot(D(y)(x) = x^2 - 2 * y(x)/2, [y(x)], x = 0.01..5, y = -4..4,$$
$$arrows = LINE);$$

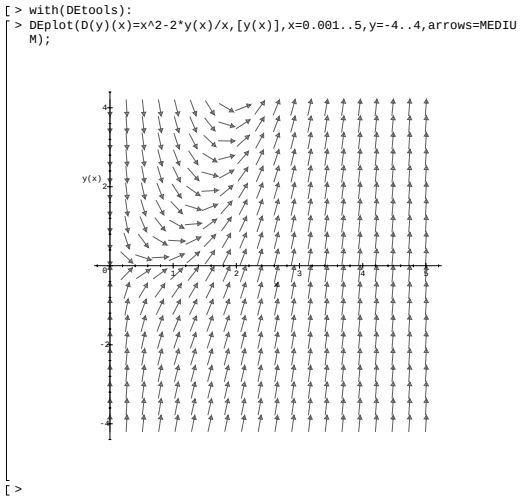


Figure 0.1: Slope field and possible solutions.