Basic Ideas of Modeling

- The differential system is a good way to model the dynamics of *interaction, interference, influence* among different *species, environmental factors, political powers, or physical quantities.*
- Consider the system

$$\begin{aligned} \frac{dy_1}{dt} &= f_1(t, y_1, y_2) \\ \frac{dy_2}{dt} &= f_2(t, y_1, y_2) \end{aligned}$$

  - ◇ The left-hand side represents the rate of change of the y variables with respect to t.
  - ◇ The right-hand side represents how the combined *current* status of y_1 and y_2 (and t) affect the rate of change of each individual variable.
  - ◇ Sometimes the independent variable t does not come into play. We call such a system *autonomous.*
- In modeling, we usually should have some ideas about how f_1 and f_2 should be *written up.*
  - ◇ The ideas could come from some laboratory empirical data, or from natural observation.
  - ◇ Sometimes the ideas come from theoretical assumptions or hypotheses.
  - ◇ The description in f_1 and f_2 usually carries some unknown parameters whose values need to be determined to best reflect the reality.

- The suitability of a model needs to be tested against observation from real phenomenon before it can be accepted as a general model.
  - ◇ It is possible that the model is overly simplified. Need to modify the model, or to develop a more complicated model.
  - ◇ It is possible that the values of the parameters are not valid. Need to re-estimate the parameters.
  - ◇ The process of designing of a model, comparing the solution from the model with real data, and re-designing usually need to be iterated several times.
- Consider the example:

$$\begin{aligned} \frac{dy_1}{dt} &= ay_1 + by_1y_2 \\ \frac{dy_2}{dt} &= cy_2 + dy_1y_2. \end{aligned}$$

  - ◇ First we study the system as is given:
    - ▷ The case, $a > 0$, $c < 0$, $b < 0$, and $d > 0$, represents that y_1 is a prey while y_2 is a predator (in a broad sense.)
    - ▷ The case, $a > 0$, $c > 0$, $b < 0$, and $d < 0$, represents that both y_1 and y_2 can survive without the other, and that they are competing.
    - ▷ The case, $a > 0$, $c > 0$, $b > 0$, and $d > 0$, represents that both y_1 and y_2 can survive without the other, and that their co-existence helps each other.
    - ▷ The case, $a < 0$, $c < 0$, $b > 0$, and $d > 0$, represents that without the other species, both y_1 and y_2 will extinct, and that they are mutually dependent.
  - ◇ Sometimes the population growth as is described above is not valid. For example, it is likely that either species *cannot* grow unbounded. There is a maximum (or minimum) population that an environment can sustain (called the logistic growth.) We may thus modify the model to become

$$\begin{aligned} \frac{dy_1}{dt} &= ay_1 + by_1y_2 - ey_1^2 \\ \frac{dy_2}{dt} &= cy_2 + dy_1y_2. \end{aligned}$$