Pendulum Models

Consider that case that a light, rigid rod of length ℓ has a mass m attached to one end and has the other end fixed to a pivot.

- Single pendulum with no friction,
  - ◇ Newton's law implies the equation of motion,

$$m\ell\frac{d^2\theta}{dt^2} = -mg\sin\theta$$

  - ◇ We can rewrite the system as a *first order* equation,

$$\begin{aligned}\frac{dy_1}{dt} &= y_2\\ \frac{dy_2}{dt} &= -\frac{g}{\ell}\sin y_1\end{aligned}$$

    where the variables are understood as

$$\begin{aligned}y_1 &= \theta\\ y_2 &= \frac{d\theta}{dt}.\end{aligned}$$

- The behavior of the pendulum can be understood from the numerical solutions. Indeed, the phase plane is also very informative. See the Maple code **lecture9.mws**.
- Single pendulum with linear damping,
  - ◇ Suppose the motion creates a torque that is opposite and proportional to the angular velocity.
  - ◇ The damped system is

$$\begin{aligned}\frac{dy_1}{dt} &= y_2\\ \frac{dy_2}{dt} &= -\frac{g}{\ell}\sin y_1 - ky_2.\end{aligned}$$

- Single pendulum with constant load,
  - ◇ Suppose a constant force L (in one direction) is applied to the pendulum.
  - ◇ The loaded pendulum satisfies

$$\begin{aligned} \frac{dy_1}{dt} &= y_2 \\ \frac{dy_2}{dt} &= -\frac{g}{\ell}\sin y_1 - ky_2 + L. \end{aligned}$$

  - ◇ Note that the equilibrium point get *shifted* because of the constant load.