Double Pendulum

- Consider a double pendulum where a simple pendulum has a second identical pendulum suspended at its end.

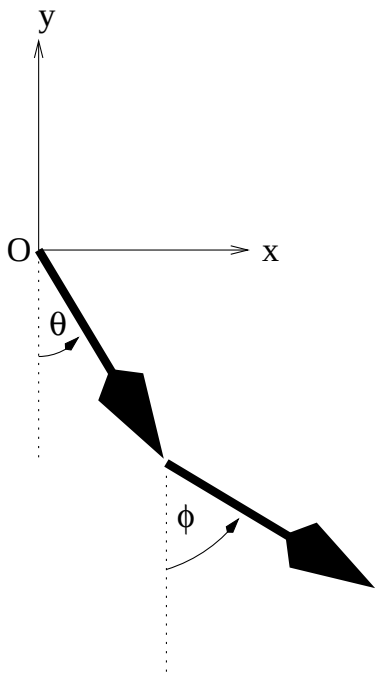


Figure 0.1: A double pendulum

- Let the motion be measured by the two angles θ and ϕ with respect to the downward vertical. Define

$$\begin{aligned} y_1 &= \theta, \\ y_2 &= \frac{d\theta}{dt} \\ y_3 &= \phi, \\ y_4 &= \frac{d\phi}{dt}. \end{aligned}$$

- Note that the two angles can be changed independently. The equation of motion is very complicated.

$$\begin{aligned}
\frac{dy_1}{dt} &= y_2; \\
\frac{dy_2}{dt} &= \frac{-y_2^2 CS - y_4^2 S - 2\omega^2 \sin y_1 + C\omega^2 \sin y(3)}{2 - C^2}; \\
\frac{dy_3}{dt} &= y_4; \\
\frac{dy_4}{dt} &= \frac{2y(2)^2 S + y_4^2 SC + 2C\omega^2 \sin y_1 - 2\omega^2 \sin y(3)}{2 - C^2}
\end{aligned}$$

  where

$$\begin{aligned}
C &= \cos(y_1 - y_3), \\
S &= \sin(y_1 - y_3).
\end{aligned}$$

- The system can be totally chaotic. See the file **movie_2_pendulum.m**.