Stiff Systems

- When two or more dependent variables in a differential system change at a wildly separated time scale, i.e., when one variable changes much more rapidly than the other variable, we have what is called a *stiff system*.
  - ◇ Stiff systems are difficult to integrate:
    - ▷ To track the fast changing solution, small step sizes must be used.
    - ▷ To track the slow changing solution, the integration has to be done over a large interval of length.
  - ◇ To solve stiff systems, it is generally believed that the method must be *implicit*.
    - ▷ RKF45 is an explicit method.
    - ▷ The backward Euler method $y_{n+1} = y_n + hf(x_{n+1}, y_{n+1})$ is an implicit method.
    - ▷ To get y_{n+1} from an implicit method, a nonlinear equation solver must be used. This means extra cost in addition to programming complexity.
- Stiff problems occurs in applications such as chemical system and nuclear reaction.
  - ◇ There are special numerical methods designed for solving still problems, but there is no clear champion method for all problems.
  - ◇ **ode15s** and **ode23s** are two MATLAB routines for solving stiff problems.
    - ▷ The call to **ode15s** and **ode23s** is exactly the same as that to the new **ode45**.

$$[tout, yout] = ode15s('chemistry', tspan, y0)$$

    - ▷ We can illustrate the difference in the performance between a stiff solver and a non-stiff solver.