Conclusion

- Many natural phenomena can be modeled by differential equations. These differential equations can be solved by numerical methods. In this class, we have learned several numerical methods for solving *initial value problems in ordinary differential equations*.
- The basic ideas of a numerical ODE method can be understood from the Euler method.
  - ◇ It shows how a numerical method can be formulated from the Taylor's series expansion.
  - ◇ It shows how the past values can be used to advance to the next step.
  - ◇ It show how the initial values are used to start out the iteration.
  - ◇ It motivates the need of a variable-step method.
- The family of Runge-Kutta methods form a much more sophisticated class of methods.
  - ◇ The idea of introducing the complexity is to derive higher order numerical schemes.
  - ◇ Many years' research seems to boil down a few selected methods among which the RKF45 is the most famous.
  - ◇ The RKF45 formula is not only efficient, but also provides an error estimator that can be used to adjust the step size adoptively.
- There is a class of problems that cannot be solved by any ordinary Runge-Kutta method, say the RKF45.
  - ◇ The iteration will become stalled by the so called stiffness of the problem.
  - ◇ Stiffness is intrinsic in the problem. Users usually cannot tell stiffness until it occurs.

- ◇ Stiff problems need stiff solvers.

- We did not learn any programming details about these numerical methods. Instead, we use existing software packages. In particular, we use the following routines available from MATLAB:

  - ◇ **ode45.** This code uses precisely the RKF45 formula.
  - ◇ **ode15s** or **ode23s.** These two codes are more expensive and are designed for solving stiff systems.

- A typical call to any ODE solver requires several fundamental information:

  - ◇ Initial values.
  - ◇ Local error tolerance.
  - ◇ Description of the vector field.
  - ◇ The way the data are collected.

- Future development:

  - ◇ The next generation MATLAB (5 or higher) will have all the codes available.
  - ◇ FORTRAN codes are also available from network or commercial packages.
  - ◇ If you want to know more theory, take MA427-428 for general introduction to numerical analysis, MA529-530 for graduate level numerical analysis, or MA583 for special topics for numerical ordinary differential equations.