Gaussian Quadrature

- Newton-Cotes quadratures for the integral $I(f) = \int_a^b f(x)dx$ are based on the integration of the polynomial $p(x)$ that interpolates $f(x)$ at a set of pre-selected nodes in $[a, b]$.
  - ◇ The weights of a Newton-Cotes are determined by $\omega_j = \int_a^b \ell_j(t)dt$.
  - ◇ It can be proved that the degree of precision for a Newton-Cotes formula of *equally spaced* nodes $x_0, x_1, \ldots, x_n$ is
  - ◇ $n + 1$, if n is even (such as Simplson's rule).
  - ◇ n, if n is odd (such as the trapezoidal rule).
- Gaussian quadratures adopt a different approach in which both the abscissas x_i and weights α_i are to be determined *simultaneously* so that the quadrature

$$Q_n(f) = \sum_{i=1}^{n} \alpha_i f(x_i) \tag{1}$$

  has a maximal degree of precision.
  - ◇ Since there are $2n$ unknowns in (**??**), the requirements

$$E_n(x^k) = 0, k = 0, 1, \ldots, 2n - 1 \tag{2}$$

    supply $2n$ equations.
  - ◇ It is expected that the maximal degree of precision is $\geq 2n - 1$.
- To determine the Gaussian quadrature, one approach is through the orthogonal polynomials.