# Predict-Correct Methods

- A predictor-corrector (PC) method consists of two methods:

  ◇ An explicit method is used to make an initial guess of the solution.

  ◇ An implicit method is used to improve the accuracy of the solution.

- Two way to process a PC method:

  ◇ Repeat the correction until no more improvement can be made, say, a preassigned tolerance $\|y_{n+1}^{(s+1)} - y_{n+1}^{(s)}\| < \epsilon$, is achieved.

  ◇ Fixed the number of corrections such as a $P(EC)^m E$ method.

- Let $P$ stand for the predictor

$$\sum_{i=-1}^{p^*} a_i^* y_{n-i} + h \sum_{i=0}^{p^*} b_i^* f_{n-i} = 0.$$

Let $C$ stand for the corrector

$$\sum_{i=-1}^{p} a_i y_{n-i} + h \sum_{i=-1}^{p} b_i f_{n-i} = 0.$$

Let $m$ be a fixed integer. Then by a $P(EC)^m E$ method we mean the following scheme:

$$y_{n+1}^{(0)} = \sum_{i=0}^{p^*} a_i^* y_{n-i}^{(m)} + h \sum_{i=0}^{p^*} b_i^* f_{n-i}^{(m)}, \tag{1}$$

$$f_{n+1}^{(s)} = f(x_{n+1}, y_{n+1}^{(s)}) \tag{2}$$

$$y_{n+1}^{(s+1)} = \sum_{i=0}^{p} a_i y_{n-1}^{(m)} + h \sum_{i=0}^{p} b_i f_{n-i}^{(m)} + h b_{-1} f_{n+1}^{(s)} \tag{3}$$
$$\text{for } s = 0, \ldots, m-1,$$

$$f_{n+1}^{(m)} = f(x_{n+1}, y_{n+1}^{(m)}). \tag{4}$$

- The selection of the pair of the predictor and the corrector should not be arbitrary. It can be proved that it is futile to have the predictor better than the corrector. In practice, it is believed that the choice $r^* = r$ with $m = 1$ is the best combination.

- An example of the 4-th order Adams-Bashforth-Moulton pair with their local truncation errors;

$$
\begin{aligned}
y_{n+1} &= y_n + \tfrac{h}{24}\left(55 f_n - 59 f_{n-1} + 37 f_{n-2} - 9 f_{n-3}\right), \\
&\quad T_{n+1} \approx \tfrac{251}{720} h^5 y^{(5)} \\
y_{n+1} &= y_n + \tfrac{h}{24}\left(9 f_{n+1} + 19 f_n - 5 f_{n-1} + f_{n-2}\right), \\
&\quad T_{n+1} \approx -\tfrac{19}{720} h^5 y^{(5)}.
\end{aligned}
$$

- An important advantage of the PC method is the ease of estimating local truncation errors. Suppose $r^* = r$. Then

$$
y(x_{n+1}) \; - \; y_{n+1}^{(0)} = c_{r+1}^* h^{r+1} y^{(r+1)} + 0(h^{r+2}) \tag{5}
$$

$$
y(x_{n+1}) \; - \; y_{n+1}^{(m)} = c_{r+1} h^{r+1} y^{(r+1)} + 0(h^{r+2}). \tag{6}
$$

It follows that

$$
y_{n+1}^{(m)} - y_{n+1}^{(0)} = (c_{r+1}^* - c_{r+1}) h^{r+1} y^{(r+1)} + 0(h^{r+2}). \tag{7}
$$

Therefore, the global error $y(x_{n+1}) - y_{n+1}^{(m)}$ is estimated by

$$
y(x_{n+1}) - y_{n+1}^{(m)} \approx \frac{c_{r+1}}{c_{r+1}^* - c_{r+1}} \left( y_{n+1}^{(m)} - y_{n+1}^{(0)} \right). \tag{8}
$$

  ◇ The above error estimator is called the Milne's device.

  ◇ Note the right hand side of (??) does not involve any high order derivative calculation.

  ◇ Note also the local assumption is not realistic in practice. Thus the estimate should be used with conservation.