Shifted QR Algorithm

- The shift can be used in the QR algorithm in exactly the same way that use in the inverse power method to accelerate the convergence.
- QR Algorithm with Origin Shift:
  - ◇ Given $A \in \mathbf{R^{n\times n}}$, define $A_1 := A$.
  - ◇ For $k = 1, 2, \ldots$, iterate until convergence:
    - ▷ Select a shift factor c_k;
    - ▷ Calculate the QR decomposition,

$$A_k - c_k I = Q_k R_k;$$

    - ▷ Define

$$A_{k+1} := R_k Q_k + c_k I.$$

- Note that $R_k = Q_k^T (A_k - c_k I)$. So $A_{k+1} = Q_k^T (A_k - c_k I) Q_k + c_k I = Q_k^T A_k Q_k$. That is, A_{k+1} and A_k are orthogonally similar.
- Wilkinson's shift:
  - ◇ Suppose A_k is upper Hessenberg.
  - ◇ The shift factor c_k may be determine from the eigenvalues, say μ_k and ν_k, of the bottom 2×2 submatrix of A_k.
  - ◇ If both μ_k and ν_k are real, we take c_k to be μ_k or ν_k according to whether $|\mu_k - a_{nn}^{(k)}|$ or $|\nu_k - a_{nn}^{(k)}|$ is smaller.
  - ◇ If $\mu_k = \overline{\nu}_k$, then we choose $c_k = \Re \nu_k$.
- Experimental results show that Wilkinson's shift is highly effective.